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Art music in decline? Time for the Golden Ratio

Autor/es

JULIO EMILIO MARCO FRANCO

Director/es

THOMAS LUDWIG SCHMITT

Facultad

Escuela de Máster y Doctorado de la Universidad de La Rioja

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Art music in decline? Time for the Golden Ratio

Autor:

Julio Emilio Marco Franco

Tutor: Thomas L. Schmitt

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Abstract

The debate on the supposed crisis of art music is interpreted as an opportunity to propose new developments, focused on the combination of the traditional European aesthetic components (tonality, consonant intervals, harmonics) with new elements: the Golden Ratio (GR) and microtonality. The determinants of musical pleasure and their different combinations with GR are reviewed, focusing on the analysis of microtonal golden scales. After reviewing and comparing several designs, it is concluded that a new auric construction of 34 equally tempered tones constitutes the theoretical model that best suits the objective of combining the components of music pleasure with the GR and microtonality.

Further studies of practical development of the 34-tone scale are required to confirm its musical aesthetic possibilities.

Resumen

El debate sobre la supuesta crisis de la música culta se interpreta como una oportunidad para proponer nuevos desarrollos, centrados en la combinación de los componentes estéticos tradicionales europeos (tonalidad, intervalos consonantes, armónicos) con nuevos elementos: la proporción áurea (GR) y la microtonalidad. Se revisan los determinantes del placer musical y las diferentes opciones para su combinación con la GR, centrándose en el análisis de las escalas doradas microtonales. Tras revisar y comparar diversos diseños, se concluye que una nueva construcción áurica de 34 tonos de igual temperamento constituye el modelo teórico que mejor se adapta al objetivo de combinar los componentes del placer musical con la GR y la microtonalidad.

Se requieren ulteriores estudios de desarrollo práctico de la escala de 34 tonos para confirmar sus posibilidades estéticas musicales.

***Key words**:* 34-tone golden scale, musical scales, Golden Ratio, music pleasantness, microtonality, consonant intervals.

> In this technological Middle Ages […] inevitably music and scientism will continue to forget that to generate the sounds one needs to know the *proportio aurea* studied by giants of thought such as Pythagoras, da Vinci, Bruno and Böhme […] forgetting that relations between the notes should follow the same mathematical ratios that govern the reproduction of many species, the phyllotaxis and our solar system [...] (Tuis, 2010, p. 13 [original in Italian]).

Foreword

Is music in crisis and in need of new findings and developments? Is symphonic orchestra sustainability in jeopardy?

In case of an affirmative answer to the questions: should something be done? Who should do that “something”? What is the role for the individual musician (the artist) in that matter? In the case of accepting convenience to promote or contribute to change, what is the new framework? Are there any aesthetic elements of music consistently requested to be kept over time? Should consonant harmony be maintained? What new elements to use if any? What is the target audience for art music in the future?

The first part of this work discusses these issues, reviews the enduring essential components to achieving musical pleasantness, sets the framework, limits and conventions for this research, and elucidates on the starting points.

Once the decision to maintain a proactive role as a composer is set, focusing on wide audiences, and with consonance kept up as the key for pleasantness, the second part of this work seeks to incorporate two additional ingredients, microtonality and the Golden Ratio (GR).

The GR is a well-known constituent for art aesthetics but has not been extensively used in music, thus offering an opportunity for future developments.

The objective of this work is to answer the question: which is the best musical scale combining consonance, microtonality and the Golden Ratio?

From the very first moment it must be said that there is no proof that music composed with a GR scale will improve music pleasantness. In fact, current experiences with GR scales, until now, have been quite disappointing; however, it is also true that those experiences did not give priority to music pleasantness in their construction. Consonance was not an issue. Thus, there is a good chance to see a different result in this study.

1. Introduction: The Starting Point. Art Music Crisis, Trends and Audiences. Something Requiring Action?

> […] current crisis in classical music comes in important measure from the obsessively narrow ways we have trained musicians for more than two centuries [...] a field where there have never been many jobs but where there are now fewer each year (Freeman, 2014, p. XVII).

1.1. Art Music Crisis: Opinions and Facts

Per Booth and colleagues, every research project must face a problem (Booth, Colom, & Williams, 1995, p. 57). The arguments to start with may be old, or new, but the problem must be significant (Eco, 2001, p. 26).

One of the most critical issues in relation to the so called “classical” music is the debate about its crisis and its survival (Flanagan, 2012).

> Music’s basic grounding was crumbling rapidly, and it older vocabulary exhausted. The question was when music as previously known would dissolve altogether (Dubal, 2013, part 5: The Age of Modernism, Chapter: Arnold Schoenberg, para. 5).

Is this music becoming obsolete? Journalist Wheatcroft (1998) states:

> Classical music faces a multiple assault from greed, apathy, atrophy and a chronic lack of funding. The end of a great, centuries-old tradition would be a tragedy 'No one can really be sure what he likes' [headline].

Another report “Classical music is dead in America” (Vanhoenacker, 2014), provides striking information about the reduction and ageing of art music attendees, plus the financial difficulties that the author summarizes as: “Even if every seat were filled, the vast majority of U.S. symphony orchestras still would face significant performance deficits” [subhead]. Charlie Albright (2013) goes even further in his paper “'Classical' music is dying [...] and that's the best thing for classical music” [headline].

Reports on the classical music crisis are not only restricted to journals. The music education expert Robert Freeman comments on a narrow way of training musicians: “The message of my book is that the current crisis in classical music comes in important measure from the obsessively narrow ways we have trained musicians for more than two centuries” (2014, p. XVII). The same idea of narrowness, or constriction is present in the works of other authors (Born, 2010; Cross, 2015; Davidson, & Edgar, 2003). Classical music is criticized for its obsolescence, out of touch with multimedia and digital developments,

interdisciplinary artistic manifestations or interactive performances, while keeping away from other cultures and pitches.

Some talk about the crisis of musical culture or values (Johnson, 2002; Weiner, 1986), others find that the crisis has even extended to religious music (Wheaton, 2005). There are books talking about the crisis evolution (Horowitz, 2005), market reinterpretation (Jeong, 2016) and, of course, about the crisis in music education (Dickinson, 2013; Freeman, 2014).

Available data for Spain (Fundación SGAE, 2017, pp.18-20), the UK (Moss, 2007, para. 4), the USA (National Endowment for the Arts [NEA], 2009, p. 3), Australia (Australian bureau of statistics, 2010, p. 10) and other places (Flanagan, 2012, p. 156; McClintock, 2017, [headline]; Rosen, 2011) show a decline in revenues, aging audience and financial difficulties.

> Live orchestral music is essentially a charity case. A Bloomberg story on the recent wave of orchestra bankruptcies (an unheard-of phenomenon outside of the U.S., says Flanagan) notes that by 2005, orchestras got more money from donations than from ticket sales (Vanhoenacker, 2014, para.7).

Per Robert J. Flanagan:

> In country after country, larger proportions of the population attended classical music concerts in the 1980s and 1990s than in early years of the 21st century…when comparisons over time are possible, attendance rates declined. Foreign countries also have similar demographic patterns of concert attendance […] Rumors of growing audiences of younger listeners in some countries remain unconfirmed. (Flanagan, 2012, p.156).

But every activity has an evolution, frequently cyclic. Albright talks about music rebirth as “true classic music” (2016, para. 19), and Tonkin (2015, [headline]) mentions the endless cycle of self-reinvention. A cyclic influence in taste is, however, denied by Flanagan (referring to the NEA figures); the author points out:

> More important for the question of preferences, the percentage of adults who report liking classical music has declined steadily since 1992 and by 2008 was below the 1982 level. This information contributes to an understanding of the trend decline, since cyclical influences on the evolution of tastes seem unlikely (2012, pp. 55).

1.2. Is Artist Action Required?

Robin (2014) and Belting (1984, 1994) have a different approach to the critical and morbid vision so far depicted for future of art music. While it is true that figures show that large orchestras’ financial sustainability is not easy to achieve, and that classical music audiences are declining and aging, that is true only in West and not in Asia (Huang, 2011; Paarlberg, 2012).

Moss also presents some encouraging data:

> The Hallé, after years in the doldrums, has bounced back since Mark Elder became music director in 2000. It gives up to 70 concerts a year in Manchester - selling, on average, 70% of the tickets for the 2,400-seat Bridge- water Hall - and another 30-plus outside Manchester (2007, para.5)

The market for events and related activities is “generally considered buoyant” (Berridge, 2007, p.48). So, if there is an interest in leisure activities, why would this be different for art music?

Even accepting that a period of art music is over and exhausted, that does not mean that music itself has ended. Maybe it is time to start planning with novel approaches.

So, is action required after all these opinions and figures? It may be that art music is for a minority. Is there a need to care about the public? Neither musicologists nor music historians have invested much interest in listeners (Obelkevich, 1989).

So, again, should something be done? In case of an affirmative answer, then *who, what, for what purpose*? If we turn our eyes to political representatives as a motivating gear of that “something”, should we, then, accept that musicology is a political act? (Bohlman, 1993). Should politicians take care of how to understand and teach music? Should they put stock in how to become a “good listener?” (Obelkevich, 1989).

Such a political approach could be situated not far from the Marxist postulates of art for the people:

> To serve the interest of the working classes, such as the Avant-grade groupings immediately after the Russian revolution of 1917 and some of their successors within the Western world-the John Reed Clubs and the American Artists Congress in the United States (Supples, 2014, p. 236).

This brings to the arena the relationship between art and politics (Edelman, 1995; Hensley, 2011), particularly when the activity is subsidized – as often happens in the case of art music – with public funds, and the criteria used for its distribution. Every musician surely will have his/her own position. This text is written with a starting point – a belief that musicians *must do something* from their own side – either in composition, in performance or in any other aspect, according to individual criteria and interests. Scholars must move forward with or without crisis.

From the composer side, developing microtonal music is a promising line of action now being adopted by some musicians (Fox, 2003; Loy, 2006). It helps to keep away the idea that each new art music composition has some similarities to a previous work. Microtonal societies such as the one in Boston or the International Ekmelic Music Society are growing, and new scales and notations are in development (Marco-Franco, 2017a, 2017b). Microtonality could be combined with new performances and combined art forms.

In summary, musical crisis echoing may be, rather than a death knell, a call for incorporating current trends and technologies in the everchanging world of culture and art. It is up to the artist to decide what attitude to take in this regard.

The author's response to that call is this work focused on joining microtonality, pleasantness and a new element, the GR, searching for the answer to the question: Is there an objective way to optimise all three elements?

2. Context, Constraints and Agreements

2.1. A Question on Taxonomy: Keeping Scores

Western classical music is a poorly defined term. Per Johnson “there is no single definition of what constitutes classical music” (2009, Preface, Chapter: What is Classical Music, para. 2). The Grove Dictionary of Music and Musicians has no specific entrance for it. The closest term is art music, defined as:

> One of three categories in a contemporary taxonomy that typically includes folk, art, and popular musics. The viability of any of these categories depends upon its perceived distinctiveness from the others, although insisting upon too rigorous a separation of types runs the risk of ignoring their intersections and overlaps, and denying their numerous commonalities (Von Glahn, & Broyles, 2012).

However, this is not a taxonomy project, and some agreement about the terms is needed. After accepting the many limitations of the concept, art music will be used for the above-mentioned music with European roots usually called (Western) classical music.

This means that the term refers to music notated in some way in a stave. Scores will be available and reproduced with orchestral (not necessarily standard) instrumental timbres.

2.2. Setting Pleasantness Concept

The next elusive term is music “pleasantness”. The study of the emotional responses to music is a fascinating area, but the question of pleasantness is rather complex (Meyer, 1956; Müller, Höfel, Brattico, & Jacobsen, 2010; Narmour, 1990).

> […] people derived aesthetic pleasure from music because the musical form developed in accordance with their expectations […] expectation plays an important role in music, and how could people have expectations about musical form if not on the basis of the forms they had previously encountered? (Cook, 1987, p 10).

Even the same music, in the same person, could evoke different feelings over time (Francès, 1988; Hallam, Cross, & Thaut, 2016; Jourdain, 1997). Following Descartes's answer to the question, Tatarkiewicz translates that the reasons why a majority has a positive aesthetic feeling are “absolutely subjective and impossible to determine” (1991, p, 466).

Again, a certain contextualization is needed. Since this project aims to attract new audiences (see next section), it is necessary to agree in some way on the concept of pleasantness in music. The Oxford dictionary defines pleasant as “giving a sense of happy satisfaction or enjoyment” (Stevenson, 2010, p. 1363). Therefore, for this project, musical pleasantness will be defined, by agreement, as music that produces in its first audition a feeling of happiness, satisfaction or enjoyment in most listeners.

2.3. Selecting the Target Audience

At this point comes another debatable question: Why art for everyone? Must art creators compose for a minority selected public, or for the majority? Setting this point is important, as target audience conditions the type of music to compose and, eventually, even the scale to be used.

Again, there is debate. Arnold Shoenberg quoted: “if it is art, it is not for all, and if it is for all, it is not art” (1950, p.51). A complete opposite point of view is maintained by British artist Tracey Emin. When she was made a Commander of the Order of the British Empire, she declared: “I think that art's [sic] for everybody and everybody's entitled to the best culture, the best literature, the best education, the best that everyone can have” (Jones, 2013, para 5).

The question is on the internet, in blogs (Bond, 2009) and in comments proclaiming the mental health benefits of art for all (Alban, [n.d.], Section Creating Art Relieves Stress, para. 11).

Therefore, the decision to promote music pleasantness for a majority is questionable. Other options could be academically maintained as elegantly as this one. The target audience, which has been commented on, is not directly related to the objective of this work and is therefore not detailed at this time.

In *summary,* this project focuses on music pleasantness for much of the audience, particularly the young. These boundaries may be debatable, but they have been chosen after careful analysis of options and alternatives.

The next chapter will focus on conventional aesthetic elements, mainly harmony and consonance, and the Golden Ratio as essential cornerstones for this work.

3. State of The Art

3.1. Music Pleasantness Theories. Consonance

> It is plain that the apprehension of a melody consists in noting with both the ear and intellect every distinction as it arises in the successive sounds – successive, for melody, like all branches of music, consists in a successive production. For the apprehension of music depends on these two faculties, sense-perception and memory […] (Aristoxenus, ca. 320 BCE, as translated in Macran, 1902, pp. 193-194).[1]

As already commented, music pleasantness is difficult to define and, consequently, the contributing elements are also debatable. In the case of assuming expectations as a key element, where are these expectations coming from? It is beyond the scope of this work to expand on the expectation theories in music. Pearce, & Wiggins (2012) have extensively analysed this field. The next paragraphs will summarize the two current hypotheses about the roots of music pleasantness.

3.1.1.-The evolutionary music hypothesis. In accordance with the evolutionary music hypothesis, tonality, pentatonic scale, octave and basic consonant intervals are deeply rooted in pre-historic times, and should be considered universal values (Ambrazevičius, & Wiśniewska, 2009; Castellano, Bahrucha, & Krumhansl, 1984; Higgins, 2012; Honingh, & Bod, 2011; Kessler,

[1] The quote is incorrectly noted as pp. 102-103 in the Grove Music Online.

Hansen, & Shepard, 1984; Podlipniak, 2015; Tillmann, Bharucha, & Bigand, 2000). Also, followers of this hypothesis consider that listeners can identify the tone, even in music from culturally remote environments (Castellano, et al., 1984; Wright, Rivera, Hulse, Shyan, & Neiworth, 2000). Per Podlipniak, our ancestors could imitate and memorize sequences of pitches sung by other members of their social groups, becoming, most likely, a part of the social and religious rituals. This is considered a cognitive trait (Snyder, 2000). The cognitive implicit tonality learning process is supported by other studies (Cuddy, 1993; Huron, 2015; Krumhansl, & Agress, 2008; Tillmann, et al., 2000). Followers of the evolutionary hypothesis find support for their conclusions in prehistoric instruments (Fink, 1970; Fountain, 1999; Hickmann, Eichmann, Laufs, & Kilmer, 2000; Krause, 2012).

However, it is impossible to know for sure when music was born (Walin, Merker, & Brown, 2000). Not only is there no conclusive proof that the just pentatonic scale is universal nor even that the instruments found are really instruments in the sense we currently give to the term. From later cultures, with pictorial remains of musical instrument usage, such as Old Egypt, “there is no direct evidence of the character of the scales and tunings” (Fletcher, 2001, p 87).

The evolutionary vision remains nearly as a hypothesis that seems to have some European bias; the universal value for tonality, the just pentatonic scale and the octave are difficult to fit in with the many alternative instruments, tunes or pitches, existing in the diverse cultures over the planet (Fletcher, 2001; Hewitt, 2013; Raine-Reusch, 2008).

3.1.2.-Tonality in European and Europeanized societies. Gradual development in harmony and tonality has been attributed to cumulative exposure to tonal music, with progressive incorporation of rules and principles (Costa-Giomi, 2003; Plantinga, & Trehub, 2014).

> Repeatedly hearing things which one likes is pleasant and need not be ridiculed. There is a subconscious desire to understand better and realize more details of the beauty (Schoenberg, 1950, p 55).

It has been proven that children between 5 and 10 years are able to identify familiar melodies presented either: intact, wandering with the same key, or wandering outside the key (Dowling, 1991, [n.p.]). Edwin Gordon has worked extensively in the field of music appreciation and understanding. He developed

the audiation concept ─a way of musical thinking─ with 7 types and 5 stages and has proved children's tonal awareness and anticipation (Gordon, 1984a; Gordon, 1984b; Gordon, 2003). Other research supports these conclusions on tonality and preference for consonance (Jaśkiewicz, Francuz, Zabielska-Mendyk, Zapała, & Augustynowicz, 2016; Trainor, Tsang, & Cheung, 2002; Wassum, 1980).

The need for the tonic, deeply anchored in the most profound cognitive human traits, might explain why works of composers with good training, such as Schoenberg, with excellent musical construction from a technical point of view but lacking tonality, have had so many difficulties being accepted apart from the limited circles of music experts (Schönberg,1992).

However, the concept of tonality if far from being completely elucidated.

> One basic musical concept already mentioned, tonality, is to many a vague and only partially understood phenomenon whose potentialities are as yet unknown. Tonality, along with other fundamentals such as harmony, counterpoint and modulation, are defined in a great number of varied and often contradictory ways (Salzer, 1962, p.8).

In the next section the closely related notion of consonance will be discussed.

3.1.3.- Music consonance and harmonics. Dissonance comes from the Latin *dissonantĭa*, meaning two sounds. The idea of this duality and the "rejection" of one sound by the other was defended by Rousseau (Masters, & Kelly, 1998, p. 385).

> The understanding of consonance and dissonance as they appear 'in Western music has been beset by such notorious difficulties and entanglements as have proven the despair of music theorists. Many have abandoned hope of ever explaining to general satisfaction why musical harmony sounds harmonious (Cazden, 1980, p. 123).

Per Werner, Fay, & Popper (2012, p. 6) linkage to old auditory brain stem pathways and the social processing centres, may be one of the reasons for the ubiquity of music cross-culturally.

Consonance is essential for tonality. In the 17th century, Jean-Baptiste J. Fourier found a representation of periodic waves as the sum of sines and cosines of frequencies that are integer multiples of a single frequency (the fundamental); these waves are called harmonics (Fan, 2010). According to Helmholtz (1954) two pitches are consonant if their partials (sine waves) are aligned, or else, sufficiently far apart so as not to interfere with each other.

> When the two tones produced are too far apart, the vibrations excited by both of them at once in Corti's organs are too weak to admit of their beats being sensibly felt, supposing of course that no upper partial or combinational tones intervene (Helmholtz, 1954, p. 172).

This has been related to the anatomical properties of the ear's basilar membrane processing the signal (Tymoczko, 2011, pp. 61-62).

But as usual in many scientific disciplines, the coincidence of two phenomena does not allow us to consider them cause and effect. For example, vehicles may stop at the red light, but do not stop because the light stops them; the driver stops the car according to traffic regulations, which could have chosen another colour. It is not the red wavelength that stops the vehicle, although there is a clear and statistically ascertainable relationship between its appearance and the vehicle's halt. Similarly, could be considered for the physical wave-shape characteristics association to pleasantness. Pigmies, for example, enjoy using dissonant sounds and harmonies (Agabu, 2016, p. 270), as they will be considered in accordance within the classical (Eurocentric) conception of Helmholtz and successive authors' sound physics analyses.

Plantinga and Trehub (2014, p. 44) agree with Werner, et al., (2012, p. 235) in asserting that the origin of Western preference for consonance remains unresolved, but their experiments go against innate preferences for consonant stimuli. 6-month-old infants exposed to different consonant/dissonant pairs of stimuli selected the first as more familiar stimulus whether consonant or dissonant (Plantinga, & Trehub, 2014, p. 40).

Therefore, it seems undeniable that there is a factor of acculturation in this matter. In other words, that the "sufficiently separated limit" as stated by Helmholtz is not established universally and fixedly.

A number of methods have been considered in consonance analysis starting with matching to small integer ratios, simpler smother waveforms or less complex interval patters; interactions of nearby tones and filters (Helmholtz, 1954); number of completing pitches; unity of perception (Stumpf, 2012); sensory or peripheral beating (Helmholtz, 1954; Huron, 1991; Terhardt, 1974); expectation context; pitch fusion/tonal fusion (Stumpf, 2012); number of voices ("chorus effect"), shrillness (high frequency partials); presence of noise (conjecture), etc., (Huron, 1991). Dissonance of pure tone dyads have been studied by Plomp, & Levert (1965) taking as reference the frequency separation.

Among the several techniques to investigate consonance, some, such as the triadic comparison, are time consuming (Van de Geer, Levelt, & Plomp, 1962, p. 308). For these authors, “the musicological meaning is a different one; in musical theory the consonants are always prime, octave, fifth and fourth, irrespective of aesthetic considerations” (1962, p. 318).

As a conclusion for this section the different consonance hypotheses and theories do not reach to a consensus. If the evolutionary theories were true, music should not have great differences among different countries, unless a cultural adaptation has been made. Therefore, the acquired factor seems more or less to be a must in any case. The fact that new experiments show non-preference for tonality in children (Plantinga, & Trehub, 2014) or in isolated natives as recently found in Tsimane (McDermott, Schultz, Undurraga, & Godoy, 2016) supports the conclusions of these authors:

> The results indicate that consonance preferences can be absent in cultures sufficiently isolated from Western music, and are thus unlikely to reflect innate biases or exposure to harmonic natural sounds. The observed variation in preferences is presumably determined by exposure to musical harmony, suggesting that culture has a dominant role in shaping aesthetic responses to music (2016, p. 547).

In any case, for the purposes of this work, these hypotheses are not relevant. This research is a qualitative and quantitative break with (but not ignorant of) the historical past. As indicated, it has been designed in the framework of art music.

Composers in the future could choose either to compose within a tonal framework or not, but it is enough to look at the music industry figures to verify that tonal sonority and consonance are preferred by the public (Reportlinker, 2017).

Consequently, this work assumes that the current public, within the context of music with European roots in which this work is framed, prefers consonance as an element of pleasantness, and this research moves ahead with this premise.

3.2. The Golden Ratio (GR) [2]

3.2.1.-The Golden Ratio nomenclature and denotation. Although the name of the Golden Ratio (GR) is relatively recent (Cook, 1914, p. 472), the proportion had already been noticed by Ancient Greek geometers. Dividing a line into two fragments, one bigger (a) and one lesser (b), the geometers found that there is a point to cut the line, such, that the ratio of the whole line (a+b) to the bigger segment (a) equals the ratio of the bigger (a) to the lesser (b):

$$\frac{a+b}{a} = \frac{a}{b}$$

This quotient is the Golden Ratio (φ), and was originally discovered as a ratio, not as a number, with two possible values fitting in.

As both results τ (-1.618…) and φ (0.618...) represent the same proportion, out of the mathematical world they are frequently named indistinctly as Phi (φ) being customary the case, in the fine arts arena (Dunlap, 1997; Livio, 2013; Meisner, 2013; Pacioli, & da Vinci, 2014 [1509]).

The GR is closely related to the Fibonacci's sequence (0,1,1,2,3,5,8,13...), a progression of integer numbers where each element is the sum of the two preceding ones. Consequently, both are so closely related (figure 4) that any construction including Fibonacci sequence also includes the GR (Dunlap, 1997, p 92.)

3.2.2.-Studies of the GR through the ages. Pythagoras (ca. 570- ca. 495 BCE) chose the pentagonal star for his school. The GR has been studied and analysed from different disciplines such as geometry, natural sciences, archaeology, architecture, arts, ceramics, religion, literature and marketing with varied points of view.[2] It seems to be present from Venus of Hohle Fels around 38,000 years old, to modern company logos and commercial designs (Dunlap, 1997; Livio, 2013).

The ubiquitous presence and the supposed general aesthetic preferences for it (Angier, 1903; Green, 1995; Haines, & Davies, 1904) have also aroused

[2] Please refer to the annexes for details and images.

some questioning (Brownlee, 2015; Falbo, 2005; Fischler, 1978; Markowsky, 1992).

3.3. The GR in Music

There are basically two approaches to including the GR in music: either to use it with a conventional equal tempered 12-tone scale (12-TET) or to use a GR scale.

3.3.1.- GR using equal tempered scales (figures 62-64)**.** The existence of GR in art music ─ and the corresponding analysis from Sabaneev─ is summarised by Olsen as follows:

> The golden section has been used by composers from Dufay […] to Bach, Bartok, and Sibelius, as a way of structuring a work of music. Russian musicologist Sabaneev discovered in 1925 that the golden section particularly appears in compositions by Beethoven (97% of works), Haydn (97%), Arensky (95%), Chopin (92%, including almost all of his Etudes), Schubert (91%), Mozart (91%), and Scriabin (90%) (Olsen, 2006 p. 8).

Even authors showing considerable criticism about GR in music had to recognize its presence, at least in some cases; Madden recognizes, in his conclusions about the romantic period, that GR is present in Mendelssohn's symphonies No. 3 and No. 4, although he does not agree in relation to Chopin's prelude No. 4 (2005, p. 247).

Erik Satie *Trois sonneries de la Rose et Croix*; Debussy *Dialogue du vent et de la mer;* Mozart *Sonata No. 1* in C Major; Bartók *Music for strings, percussion and celesta*, first movement; and the beginning of the Beethoven *Fifth symphony* are, per Angelica Frey, compositions using the GR (2015).[3]

In compositions using equal tempered scales, the GR is to be included, basically, in metric, progressions, symmetry and similar elements, since the scale is usually not golden (Mongoven, 2010, pp. 127-138).

The GR has also been used in contemporary music. That is the case of the rock group Tool and the 2001 composition *Lateralus.* Even the syllabic progression of the lyrics has been analysed in that way. Also, it is possible to find recorded music with note succession following the Fibonacci digit sequence.

3.3.2.-GR and computerised music. There are some works about Fibonacci computers (Stakhov, & Olsen, 2009 p. 416), and, even, the application

[3] Sample analysis figures from Bartók, Debussy and Beethoven can be found in the annex.

of stochastic process to music has been reported, including Markov chains (Jones, 1981). Just to mention as an example, only with Zipf's laws, over 40 metrics have been tested in music (Manaris, et al., 2005).

One relevant case of joint usage of the computers and music is the architectural work of Iannis Xenakis (1990).

Stria is a famous work from John Chowning. He began the project as far back as 1972, using DEC-PDP-10 computer and SAI language, but without the possibility of further experiments he returned to Stanford (Chowning, 2007).

3.4. Building Scales

Creating new scales should benefit from a preliminary comment on musical scales history and context, but that is out of the scope of this work. Suffice is to say that the *Scale Omnibus* contains 1,018 names corresponding to 392 different scales. In Asia, there are scales for days, seasons and even for mood states (Balena, 2014). Also, it is not possible to extend this over the concepts of schisma, Phytagorean comma, major limma, Phytagorean limma and diaschisma that can be found elsewhere (Benson, 2007, p. 171).

One of the fundamental problems in relation to building scales (either golden or not) is the fifth problem.[4] It is simply as when we want to find a number odd and even at the same time. An integer number of fifths never matches with any other integer number of octaves (Loy, 2006). The twelve-tone scale (12-TET) is a compromise. It was only in late 19th century when its current division of the octave in 1200 cents ─with the 100 cents equivalent for a tempered semitone─ was set (Ellis, 1880).

The arguments in favour of different numbers of notes in the octave or whether the intervals are to be equal or not have given rise to varied discussions and approaches. However, it must be said that the number of notes is a key element in terms of consonance (Cartwright, González, & Piro, 2009, p. 184).

Another concept, also from 19th century, is the Savart. It represents a logarithmic measure of tuning, the shortest musical interval an averaged trained ear can recognize. After the Wood modification, the octave contains exactly 300 Savarts (25 in a halftone); this means that the smallest recognizable interval is 4

[4] $2^x=3^y$ has no solution for integer numbers.

cents (Wood, 1944). Cents and Savarts are two ways to indicate an interval between two pitch frequencies (Hz).

GR scales face an additional problem. The GR is irrational and, again, does not fit directly into the consonant intervals based in simple natural numbers. A compromise about what to keep as priority (partials, harmonics, triads, consonant intervals) is needed. Whether it will be equal-temperament or not is another decision to make.

In summary, the GR has been present as an aesthetic element throughout time and cultures across the planet. In music, it has been used mainly in structure. The GR (irrational value) based scales have theoretical problems to fit in with the European aesthetic music parameters (based on simple rational consonant intervals). This work focuses on the challenge of combining the GR with consonance, searching for the best GR consonant, equal tempered scale.

4. Purpose, Goal and Objective

The purpose or reason for this work is to expand art music to laymen and young people, in a framework of economic sustainability.

The goal to accomplish this is to join the basic music aesthetic elements, mainly consonance, with microtonality and the GR.

The objective or short-term plan of this work is to answer the question: which is the best musical scale that objectively incorporates, consonance, microtonality and the Golden Ratio?

As a *sub-objective*, the compatibility with the common equal tempered scale of 12 tones (12-TET) will be analysed.

5. Methodology

5.1. Sources

Background material has been acquired from the National Research Council of Italy (Bologna unit), Majorca University (UIB), La Rioja University (Unirioja) and Aveiro University library resources and databases. Online information has additionally been searched through E-libraryUSA (US Department of State), Jstor, Scribd, and directly on the internet. A stay abroad with the Bologna unit of the National Research Council of Italy was procured for

a better information about scales construction and details. The scale of 34-TET-GR has been conceived as result of this stay.

5.2. Procedure Analyses[5]

The working plan has included evaluation of the following items in the scales found, based on the GR: a) Equal temperament (EQ); b) Mean quadratic dispersion (MQD) value in relation to the number of notes[6] for equal tempered scales (Cartwright, González, Piro, & Stanzial, 2002, p.56); c) Golden Ratio; d) Consonance, scored in relation to Pythagorean and Just Intonation (JI) intervals; and e) compatibility with the 12-TET Scale. A Likert scale has been used for some items.

The Interval between two frequencies (Hz_a and Hz_b) has been measured in Savarts with the formula: $1000*Log_{10}(Hz_b/Hz_a)$ and in cents, using $n= 1200* \log_2 (Hz_b/Hz_a)$.

6. Results

6.1. GR Scales Found in the Research Project

The following scales related to Golden Ratio have been analysed:

- 741 (Freivald, [n.d.]; Sevish, 2017).
- Erv Wilson 22-T, 31-T, and 41-T (Finnamore, 2001; Wilson, 1975).
- Heinz Bohlen- Kess van Prooijen- John R. Pierce- 833 (Bohlen, 2012; Enrich, 2002; Pareyon, 2011; Walker, 2001).
- Golden meantone (Kornerup, 1935).
- Square root of Phi 17, 23 and 49 tones (Breed, [n.d.]; Vajda, 2008).
- Lange PhiΛ(n /7) (Lange, 2013).
- Mongoven (Mongoven, 2010).
- Cartwright, et al., 34-T-GR (2002).
- Marco, & González, 34-TET-GR (Marco-Franco, 2017a, 2017b).

[5] Please refer to the annex I for tables and details. Microsoft© Excel formulas or macros have been used for calculations.

[6] This number (σ) is the square of the difference between the note of the equal tempered scale that best approximates each harmonic interval, multiplied by the relative weight of each interval and summed over all the intervals.

6.2. Scale Patterns

Table 1 summarises the main results of the comparative analyses:[7]

Table 1

Summary of scales patterns

	ET	MQD	GR	Adjust Phy.(Cts)	Adjust Phy. (Sv.)	Adjust Just (Cts)	Adjust Just (Sv.)	Adjust 12-TET
	+/-	0-3+	0-3+	Coef.	Coef.	Coef.	Coef.	Coef.
741	0		+++	106.6	26.7	116.9	29.3	78.26
Wilson-22	0		0	21.5	5.4	0.0	0.0	23.05
Wilson-31	0		0	5.8	1.4	21.5	5.4	23.00
Wilson-41	0		0	0.0	0.0	0.0	0.0	23.66
Pierce	0		+++	177.4	44.5	177.4	44.5	122.78
Bohlen	0		+++	182.1	45.7	182.1	45.7	122.15
GR-Meantone	0		+	51.6	13.0	17.2	4.3	28.71
Sqrtphi-17	+	+	+	101.0	25.3	101.0	25.3	72.17
Sqrtphi-23	+	0	+	65.4	16.4	62.0	15.6	43.44
Sqrtphi-49	+	++	+	24.5	6.1	19.1	4.8	16.87
LangePhiΛ(n /7)	0		+	1357.4	340.5	380.8	95.5	Not applicable
Mongoven	0		++	Not applicable	Not applicable	Not applicable	Not applicable	Not applicable
Cartwright	0		+++	31.4	7.9	0.0	0.0	27.89
Marco	+	+++	+++	35.3	8.9	11.8	3.0	30.37

Note. ET: Equal temperament (absent or present). MQD value: +3 for σ values up to three, +2 for values between four and six, +1 for values between seven and ten and 0 for σ over ten. GR: Score is 0 if absent, 1+ for GR inclusion through mathematical complex logarithmic, exponential, squaring functions or an irregular use of GR through the scale, 2+ for inclusion of GR with simple mathematical functions (addition, subtraction, multiplication, division) and 3+ for use of GR or the Fibonacci series either directly or through ratios of convergence. For Pythagorean and JI, the adjusted sum coefficient of consonant intervals (Oct, 6M, 5P, 4P, M3, Unison) expressed in cents and in Savarts. For adjustment to 12- equal tempered the value is the sum of relative weight differences for the 12 notes.

7. Discussion

7.1. Model Selection

The 741 scale as the majority of xenharmonic scales sounds dissonant. The interval itself 741 sounds like a “horrible” sharp fifth (Sevish, 2017). The fifth has over 9 Hz difference to just scale. Coefficients are very poor for consonance or for proximity to 12-TET.

Wilson scales, include JI, so they are very consonant, but they are not tempered. Moreover, it seems that the GR does not play much of a role in these scales as the JI remains fixed (except in a partial exception for 31-T) independently of the number of tones. Erv Wilson published his works for years in the informal journal of experimental music, *Xenharmonikon*. Scales of 17, 19, 22, and 31 tones are especially pleasing, but their intervals sound strange and often with close similarities with Japanese music, something consistent with the time that the musician was living in that country (Finnamore, 2001). Some of his

[7] All pitches are referred to the IPN (International Pitch Notation) or American Standard Pitch Notation (ASPN, for octave= 4 [A4=440]).

constructions (17,19, 22 and 31 tones) had been used in practice with different instruments (Wilson, 1975).

Pierce and Bohlen scales are far from both JI and 12-TET as they are not based on octaves. The creation was done independently by Heinz Bohlen, Kess van Prooijen and John R. Pierce. Bohlen designed the first version (13th scale, just), in early 1970'. Six years later, Pierce discovered the same scale, but proposed that it be used in the equal tempered form (Walker, 2001).

As described by Bohlen, the scale is symmetrically heptatonic, and thus different from usual scales based on juxtaposition of tetrachords. Consequently, it is not based on octaves and contains no 2/1 ratios. The tritave is divided into 13 steps. Intervals are based only on odd integer frequency ratios, in contrast with the harmonic series employing both odd and even ratios. Probably, as seems to be the case, the instruments that predominantly produce odd harmonics, such as the clarinet, will have more possibilities to obtain better results with the scale. Although the author appears to be unaware of it, the basic intervals follow the Fibonacci sequence. The design has been modified in several ways, including 36 equal tones, containing the 12 conventional notes, and 39 equal tones (Enrich, 2002). Even though the scale has been presented at electroacoustic festivals, and there has been some instrument building and occasional performances, about 40 years after creation its expansion still seems to be very limited.

GR meantone scale is non-tempered, far from GR as the number is transformed but adjusts well to JI or to 12-TET except for major 3rd interval.

Out of the three square-Phi scales, the 49-T has the best parameters. It is equal tempered, MQD (σ) is reasonable (slightly over five, that is range +2), and adjusts well both to JI and to 12-TET. It has the inconvenience that the GR seems far away. Phi is a ratio and complex operations with it suppose the substantial risk of making it lose its aesthetic properties. Transformation of Phi can reach to any number. As an example, Lange (2013), indicates how to reach rational simple numbers by use of Phi-powers:

> Phi-1+Phi-2=Phi0=0,61803399+0,38196601=1,00000000
> Phi1+Phi-2=1,61803399+0,38196601=2,00000000
> Phi2+Phi-2=2,61803399+0,38196601=3,00000000
> Phi2+Phi-2+Phi0=2,61803399+0,38196601+1,00000000=4,00000000
> (Lange, para. 8, 2013).

The question here is whether it is truly a GR scale.

Still further far from the traditional aesthetic patterns are both Lange and Mongoven scales. As indicated by the later “These tunings contain no octaves – or any other pure interval for that matter” (Mongoven, 2010, para. 24).

Focusing on the elements indicated above, the 34-T-GR scale of Cartwright, et al., (2002) fits well with the conditions required, but has the inconvenience of being non-equal tempered.

With the transformation of this scale into equal temperament, the approximation to GR is maintained, fitting very well with JI and not significantly further from the 12-TET scale. This 34-TET-GR (Marco-Franco, González, 2017), scale represents the best compromise of a direct development from the GR, equal-temperament with exact octave, excellent adjustment to consonant intervals and relatively nearness to the 12-TET (that sub-objective is only important as to take advantage of already existing instruments, or in combined compositions using both scales, using appropriate interval combinations).

A notation for the 34-tone scale was patented with the Ministry of Culture in Spain and presented at the Symposium of the *International Ekmelic Music Society* in Salzburg, July 2017 (Marco-Franco, 2017).

7.2. Advantages and Foreseeable Difficulties

The 34-TET-GR scale, with such a wide a range of pitches, allows an easy approach to other cultures.

However, the development of new scales, particularly when based in microtones, has several difficulties. For example, quarter-tone scales require two pianos tuned a quarter tone apart (Loy, 2006, p. 93). New scales will mean designing and building new instruments ─ with all the economic implications ─ a body of musical work, and an interested paying audience (Loy, 2006, p. 93).

Harmony must be considered in a different way, distance between notes makes conventional chords virtually impossible to play with current keyboards.

8. Conclusions

It is a must for scholars to progress towards new frontiers either in response to the so-called art music crisis or as a natural headway.

Tonality, octave, maximization of harmonics (even and odd), and consonant intervals have been recognized as constituent elements of musical pleasantness over the ages in European and Europeanized cultures, as evidenced by audience preferences tests and by sales.

The GR is an aesthetic element present for thousands of years in many cultures all over the planet. It has not been fully exploited in music, although there are several composers who have used it, both in metric and with alternative scales ─ these two possibilities not being mutually exclusive. The difficulties in matching consonance and the other pleasant constituents with the GR may be responsible for the previous low acceptance for golden scales.

The present study has analysed different approach possibilities, concluding that the new 34-TET-GR scale (Marco-Franco, & González) is the best option found to keep both music pleasantness elements and the GR.

Practical development will be necessary in the future to confirm whether this theoretical model offers real possibilities for music pleasantness.

As a final comment, should we be always restricted to consonant intervals and harmonics? It is possible that there are anatomo-physiological reasons for it (Tymoczko, 2011), but if that were the case why are they not universal but restricted to European cultures? If the pleasantness parameters are acquired as Gordon and other researchers suggest (Costa-Giomi, 2003; Dowling, 1991; Gordon, 1984a; Gordon, 1984b; Gordon, 2003), then new intervals and harmonics may eventually also become pleasant in different environments.

Additionally, in the exploration of new frontiers it should be considered that it is now possible to control inharmonic partials with a plug-in (Sevish, 2017). Consequently, it is not difficult to think that digital instruments, in a near future, could modify harmonics and thus customize the filter of alternative notes and their partials. In that case, a new range of scales and timbres now unpleasant may have their place in the future, once modified.

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